



Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Mechanics M3 (WME03) Paper 01

Question Number	Scheme	Marks
1(a)	$\pi \int_0^1 (1 + \sqrt{x})^2 dx$	M1
	$= \pi \left[x + \frac{4}{3} x^{\frac{3}{2}} + \frac{1}{2} x^2 \right]_0^1$	A1
	$= \frac{17\pi}{6} \text{ m}^3 \text{ * including units}$	A1*
		(3)
(b)	$\pi \int_0^1 x(1 + \sqrt{x})^2 dx$	M1
	$= \pi \left[\frac{1}{2} x^2 + \frac{4}{5} x^{\frac{5}{2}} + \frac{1}{3} x^3 \right]_0^1$	A1
	$= \frac{49\pi}{30}$	A1
	$\bar{x} = \frac{\frac{49\pi}{30}}{\frac{17\pi}{6}}$	dM1
	$= \frac{49}{85} \text{ m} \text{ * including units}$	A1 *
		(5)
		(8)
Notes		
NB: Penalise missing units maximum of once per question.		
(a)		
M1	Use of $\pi \int_0^1 (1 + \sqrt{x})^2 dx$. Limits not needed. π is required.	
A1	Correct integration – limits not needed	
A1*	Correct given answer correctly obtained. Must include units. Limits must be seen (sight of substitution is not required). Accept $\frac{17}{6} \pi \text{ m}^3$	
(b)		
M1	Use of $\pi \int_0^1 x(1 + \sqrt{x})^2 dx$. Limits not needed (π 's will cancel so it may not be seen)	
A1	Correct integration – limits not needed	
A1	Correct unsimplified with or without π (may see $\frac{1}{2} + \frac{4}{5} + \frac{1}{3} - 0$)	
dM1	Correct expression with their numerator (consistent π - seen in neither or both)	
A1*	Correct given answer correctly obtained. Must include units.	

Question Number	Scheme	Marks	
2.	$F \cos \alpha = mg$	M1	A1
	$F \sin \alpha = T$		A1
	$T = \frac{2mgx}{l}$ or $T = \frac{2mg(AB-l)}{l}$	M1	
	$\frac{3}{4}mg = \frac{2mgx}{l}$	dM1	
	$AB = \frac{11l}{8}$	A1	
		(6)	
Notes			
M1	Resolve vertically or horizontally, correct no. of terms, condone sign errors and sin/cos confusion (or use trig on a right-angled triangle of forces)		
A1	Correct vertical equation		
A1	Correct horizontal equation (A2 for $T = mg \tan \alpha$ from triangle of forces)		
M1	Hooke's Law. Must clearly be an extension and not AB . Since x is not defined in the question, other extensions may be used including $(AB - l)$ or xl where x is found to be the constant $\frac{3}{8}$.		
dM1	Substitute trig (not necessarily correctly) to produce an equation in 'x' (and l) only, dependent on previous M's and on having two equations.		
A1	Cao Accept 1.375l , 1.4l, 1.38l		

Question Number	Scheme	Marks
3(a)	Slant height, $l = \sqrt{\left(\frac{7a}{4}\right)^2 + (6a)^2} (= \frac{25a}{4})$	M1
	Masses Square $16a^2$ Circle $\pi\left(\frac{7a}{4}\right)^2$ Conical shell $\pi \times \frac{7a}{4} \times \frac{25a}{4}$ Total $\left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi \times \frac{7a}{4} \times \frac{25a}{4}\right]$	B1 square B1 circle B1ft (shell and total)
	Distances Square Circle Conical shell Total 0 0 2a : $\bar{x}$	B1
	$\pi \times \frac{7a}{4} \times \frac{25a}{4} \times 2a = \left[16a^2 - \pi\left(\frac{7a}{4}\right)^2 + \pi \times \frac{7a}{4} \times \frac{25a}{4}\right] \bar{x}$	M1 A1
	$\bar{x} = \frac{175\pi a}{(63\pi + 128)}^*$	A1*
		(8)
3(b)	$\tan \alpha = \frac{2a}{\left(\frac{175\pi a}{(63\pi + 128)}\right)}$	M1
	$\tan \alpha = \frac{126\pi + 256}{175\pi} \quad (\text{or} \quad \frac{2(63\pi + 128)}{175\pi})$	A1
		(2)
		(10)
Notes		
(a)		
M1	Use of Pythagoras (unsimplified). May be seen on the diagram.	
B1	Mass/area of square	
B1	Mass/area of circle	
B1 ft	Mass/area of conical shell and total. A common error is to use 6a as slant height, only ft on their calculated slant height. May derive conical shell formula from area of a sector.	
B1	All distances correct	
M1	Dimensionally correct moments equation. Must have correct number of terms including an attempt to subtract the circle. Condone a slip with an 'a' in one term.	
A1	Correct equation (no ft)	
A1*	Given answer correctly obtained. Condone missing brackets from denominator and terms reversed.	
(b)		
M1	Allow reciprocal. Must use 2a and given $\bar{x}$.	
A1	Cao Exact fraction required.	

Question Number	Scheme	Marks
4(a)	$a = v \frac{dv}{dx}$	M1
	$= \frac{3}{2} (2x+1)^{\frac{1}{2}} \times 2 \times (2x+1)^{\frac{3}{2}} = 3(2x+1)^2$	A1
	$3(2x+1)^2 = 243$	M1
	$x = 4$	A1
		(4)
4(b)	$(2x+1)^{\frac{3}{2}} = \frac{dx}{dt}$ OR $a = 3v^{\frac{4}{3}} = \frac{dv}{dt}$	M1 A1
	$\int dt = \int (2x+1)^{\frac{3}{2}} dx$ $\int 3dt = \int v^{-\frac{4}{3}} dv$	M1
	$t = -(2x+1)^{\frac{1}{2}} (+C)$ $3t + (C) = -3v^{-\frac{1}{3}}$	A1
	$t = 0, x = 0 \Rightarrow C = 1$ $t = 0, x = 0 \Rightarrow v = 1 \Rightarrow C = -3$ and obtain an equation in v and t only.	M1
	$v = \frac{1}{(1-t)^3}$	A1
		(6)
		(10)
Notes		
(a)		
M1	Use of $a = v \frac{dv}{dx}$ or $a = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$. Evidence of differentiation, power decreasing by 1. Should see a product of terms to imply 'use of'.	
A1	Correct differentiation	
M1	Independent. Use their result from differentiation and put $a = 243$ then solve for x	
A1	Cao If -5 is seen then it must be rejected or 4 must be clearly identified.	
(b)		
M1	Use of $v = \frac{dx}{dt}$ to obtain DE in x and t OR Use of $a = \frac{dv}{dt}$ to obtain DE in v and t	
A1	Correct equation	
M1	Separate and integrate (evidence of integration, power increasing by 1)	
A1	Correct integration, condone missing C	
M1	Use $t = 0, x = 0$ to obtain a value of C and obtain an equation in v and t only.	
A1	Cao Accept $v = (1-t)^{-3}$ or $v = \frac{-1}{(t-1)^3}$ or $v = -(t-1)^{-3}$	
	Note: No marks in (b) for use of $a = 243$	

Question Number	Scheme	Marks
5(a)	Use of cosine rule on triangle <i>APB</i> OR trig. on ‘half’ of the triangle <i>APB</i> to find one relevant angle.	M1
	Given answers correctly obtained.*	A1*
		(2)
5(b)	$T_A \cos 30^\circ + T_B \cos 60^\circ = mg$	M1 A1
	$T_A \sin 30^\circ + T_B \sin 60^\circ = mr\omega^2$	M1A1A1
	$r = a \sin 60^\circ$ (or $r = a\sqrt{3} \cos 30$ or $r = a \frac{\sqrt{3}}{2}$)	B1
	Solve for T_A	dM1
	$T_A = \frac{1}{2} m\sqrt{3}(2g - a\omega^2)$ *	A1*
		(8)
5(c)	Attempt to obtain one inequality on ω^2	M1
	Correct inequality	A1
	Attempt to obtain another inequality on ω^2 and use both to obtain answer	M1
	$\frac{2g}{3a} < \omega^2 < \frac{2g}{a}$ *	A1 *
		(4)
		(14)
Notes		
(a)		
M1	Either complete method to obtain one relevant angle.	
A1*	Correct GIVEN angles correctly obtained. Sufficient annotation/justification leading to both given answers eg Stating $\angle OBP = 2 \times \angle OAP$ alone is not sufficient – additional annotation or justification is required. Use of triangles to verify is acceptable.	
(b)		
M1	Resolve vertically, dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.	
A1	Correct equation	
M1	Equation of motion horizontally: dimensionally correct equation with correct no. of terms, condone sign errors and sin/cos confusion.	
A1	Correct equation, with at most one error. If $r\omega^2$ is never seen, this is an A error.	
A1	Correct equation	
B1	Cao If this is seen in (a) it must be used in (b) for this mark.	
dM1	Solve for T_A in terms of m, a, g and ω	
A1*	Given answer correctly obtained. Must see exactly.	
(c)		
M1	Correct use of either $T_A > 0$ or their $T_B > 0$ oe to obtain one inequality on ω^2 . Could be their expression for either Tension > 0 .	
A1	Correct inequality	
M1	Use both $T_A > 0$ and their $T_B > 0$ to form inequalities in attempt to obtain answer. Could be their expression for either Tension > 0 . Note: $T_B = \frac{3}{2} ma\omega^2 - mg$	
A1*	Given answer correctly obtained	

Question Number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - mgl$ or $mgl - \frac{1}{2}mv^2$ seen or implied	B1
	Use of EPE	M1
	$\frac{mg}{2l}l^2$	A1
	$\frac{mg}{2l}(l\sqrt{2} - l)^2$	A1
	$\frac{1}{2}mv^2 + \frac{mg}{2l}(l\sqrt{2} - l)^2 = mgl + \frac{mg}{2l}l^2$	M1
	Solve for v^2	dM1
	$v^2 = 2gl\sqrt{2}$ *	A1*
		(7)
(b)	$T = \frac{mg(l\sqrt{2} - l)}{l} = mg(\sqrt{2} - 1)$	M1 A1
	$\pm N + T \cos 45^\circ = \frac{mv^2}{l}$	M1A1A1
	$\pm N + mg(\sqrt{2} - 1) \times \frac{\sqrt{2}}{2} = \frac{m}{l} \times 2gl\sqrt{2}$	dM1
	* $N = \frac{1}{2}mg(5\sqrt{2} - 2)$	A1*
		(7)
		(14)
Notes		
(a)		
B1	Difference between KE and GPE, seen either way round.	
M1	Use of EPE formula at top or at B	
A1	Correct EPE at top	
A1	Correct EPE at B	
M1	Use of conservation of energy, with 1 GPE, 1 KE and 2 EPE terms, condone sign errors	
dM1	Solve for v^2 , dependent on previous M	
A1*	Exact given answer correctly obtained	
(b)		
M1	Use of Hooke's Law at B – this may appear in an attempted equation of motion	
A1	Correct unsimplified tension at B	
M1	Equation of motion at B horizontally with correct terms, condone sign errors	
A1	Correct equation with at most one error	
A1	Correct equation	
dM1	Sub for T and v^2 . Dependent on both previous M marks	
A1*	Given answer correctly obtained (exactly). If $N = -\frac{1}{2}mg(5\sqrt{2} - 2)$ then clear justification is required to reach the given answer eg use of 'magnitude' or modulus signs.	

Question Number	Scheme	Marks
7(a)	$T_A - T_B = m\ddot{x}$	M1
	$\frac{2mg}{l} \left(\frac{2l}{3} - x \right) - \frac{mg}{l} \left(\frac{4l}{3} + x \right) = m\ddot{x}$ or $\frac{mg}{l} \left(\frac{4l}{3} - x \right) - \frac{2mg}{l} \left(\frac{2l}{3} + x \right) = m\ddot{x}$.	dM1A1
	$-\frac{3g}{l}x = \ddot{x}$, so SHM	A1
	$T = \frac{2\pi}{\sqrt{\frac{3g}{l}}} = 2\pi\sqrt{\frac{l}{3g}}$ *	M1 A1*
		(6)
7(b)	$\frac{1}{2}l \times \sqrt{\frac{3g}{l}}$ or $\frac{1}{2}\sqrt{3gl}$ or $\sqrt{\frac{3gl}{4}}$ oe	B1
		(1)
7(c)	$\frac{3g}{2}$ or $1.5g$	B1
		(1)
7(d)	$x = a \cos \omega t \Rightarrow v = -a\omega \sin \omega t$	M1
	$-\frac{3}{4}\sqrt{gl} = -a\omega \sin \omega t$ to find t	M1A1
	Solve for t	M1
	$t = \frac{\pi}{3}\sqrt{\frac{l}{3g}}$ oe	A1
		(5)
		(13)
Notes		
(a)		
M1	Equation of motion in a <i>general</i> position, allow a for acceleration, correct no. of terms, condone sign errors.	
dM1	Use Hooke's Law to sub for the two tensions, allow a for acceleration. Extensions must be different and of the form $(d \pm x)$ where d is a multiple of l .	
A1	Correct unsimplified equation, allow a for acceleration.	
A1	Correct equation using $\ddot{x}$ for acceleration.	
M1	Use of $\frac{2\pi}{\omega}$ Their ω from their equation of motion, which must be in terms of x .	
A1*cs0	Given answer correctly obtained – this includes proof of SHM with conclusion and correct expression for the period.	
(b)		
B1	Cao Speed at O so must be positive. Unsimplified, ignore errors from subsequent 'simplifying' of surds.	
(c)		
B1	Cao Max acceleration so must be positive.	

(d)	
Main	
M1	Use of $x = a \cos \omega t$ to obtain $v = -a\omega \sin \omega t$ Substitution for a and ω is not required.
M1	Use $v = -a\omega \sin \omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $-\frac{3}{4}\sqrt{gl} = -a\omega \sin \omega t$
A1	Correct equation in t only
M1	Solve to find the required time, t
A1	Cao for required time.
ALT 1	
M1	Use of $x = a \sin \omega t$ to obtain $v = a\omega \cos \omega t$ Substitution for a and ω is not required.
M1	Use $v = a\omega \cos \omega t$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in t only, $\frac{3}{4}\sqrt{gl} = a\omega \cos \omega t$
A1	Correct equation in t only
M1	Solve to find t and then subtract from $\frac{1}{4}$ period to find the required time. $t = \frac{\pi}{6} \sqrt{\frac{l}{3g}} \Rightarrow \text{required time} = \frac{1}{4} \left(2\pi \sqrt{\frac{l}{3g}} \right) - \frac{\pi}{6} \sqrt{\frac{l}{3g}} = \frac{\pi}{3} \sqrt{\frac{l}{3g}}$ Eg
A1	Cao for required time, $t = \frac{\pi}{3} \sqrt{\frac{l}{3g}}$ oe
ALT2	
M1	Use of $x = a \cos \omega t$ or use of $x = a \sin \omega t$. Substitution for a and ω is not required.
M1	Using $v^2 = \omega^2(a^2 - x^2)$ with $a = \frac{l}{2}$ and $\omega = \sqrt{\frac{3g}{l}}$ to obtain equation in x only. $\left(-\frac{3}{4}\sqrt{gl}\right)^2 = \omega^2(a^2 - x^2)$
A1	Correct equation in x only. (Solution leads onto the first M mark in (d))
M1	Solves for t and then completes the method to find the required time. $\frac{l}{4} = \frac{l}{2} \cos\left(\sqrt{\frac{3g}{l}}t\right)$ e.g. or quarter period with sin method.
A1	Cao for required time, $t = \frac{\pi}{3} \sqrt{\frac{l}{3g}}$ oe
SPECIAL CASE where $a = \frac{1}{2}l$ is clearly stated as amplitude and consistently used in (b) (c) & (d)	
(b)	B1 $\frac{1}{2}\sqrt{\frac{3g}{l}}$
(c)	B1 $\frac{3g}{2l}$
(d)	Maximum M1 M1 A0 M0 A0